Chapter 1
Equations and Inequalities

Chapter 1

1.1 Linear Equations

Equations have equal signs and can be solved. Without the equal sign, it's just an expression to be simplified. Linear equations have either (1) solution, (2) no solution $\varnothing$ or (3) infinite solution $(-\infty, \infty)$ or R.

Ex 1

$3(2x-4)$	$=$	$7-(x+5)$	(Distribute)
$6x-12$	$=$	$7-x-5$	(Simplifying ea. side)
$6x-12$	$=$	$2-x$	(Get var. on one side and #'s on other side)
$7x$	$=$	14	(Divide)
x	$=$	2	One solution $\{2\}$ a conditional equation.

Ex 2

$3x+3$	$=$	$5+3x$	(There is no way that three times a # plus 3 can equal 3 times that same # plus 5.)
0	$=$	2	(False statement)
			A contradiction; $\therefore \varnothing$ no solution

Ex 3

$3x+4x$	$=$	$7x$	(This is an identity—any # times 7 will equal that same # times 7)
$7x$	$=$	$7x$	(True)
0	$=$	0	Infinite solutions or R all real.

If your equations contain fractions, then multiply through by the LCD—this will eliminate all fractions.

Ex 1 $\frac{5}{6}k-2k+\frac{1}{3}=\frac{2}{3}$ The LCD is 6. Times every term by 6.

$$\frac{6}{1}\left(\frac{5}{6}k\right)-6(2k)+6\left(\frac{1}{3}\right) = 6\left(\frac{2}{3}\right)$$

$$5k-12k+2 = 4$$

$$-7k+2 = 4$$

$$-7k = 2$$

$$k = -\frac{2}{7}$$

Ex 2 $\frac{2}{x-1} - \frac{4}{3x} = \frac{1}{x^2 - x}$ Find LCD.
Factor any denom. that needs it.

x(x – 1) LCD => 3x(x – 1) Times every term by this.

$$3x(x-1)\left(\frac{2}{x-1}\right) - 3x(x-1)\left(\frac{4}{3x}\right) = 3x(x-1)\left(\frac{1}{x(x-1)}\right)$$

$$\begin{aligned} 6x - 4(x-1) &= 3 \\ 6x - 4x + 4 &= 3 \\ 2x + 4 &= 3 \\ 2x &= -1 \\ x &= -\frac{1}{2} \end{aligned}$$

Be careful to check solutions—sometimes we must throw out solutions because they cause a zero in a denominator.

You try:

1. $x^2 + 6x = x(x+6)$
2. $2x - 4 = 2(x+2)$
3. $\frac{3}{4} + \frac{1}{5}r - \frac{1}{2} = \frac{4}{5}r$
4. $\frac{2r}{r-1} = 5 + \frac{2}{r-1}$
5. $\frac{3}{y-2} + \frac{1}{y+1} = \frac{1}{y^2 - y - 2}$

1.2 Linear applications

Reading Problems

0. Basic Unknown:

   A. The instructions for a woodworking project require three pieces of wood. The longest piece must be twice the length of the middle-sized piece, and the shortest piece must be 10 inches shorter than the middle-sized piece. Maria Gonzoles has a board 70 inches long that she wishes to use. How long can each piece be?

   B. If 2 is subtracted from a number and this difference is tripled, the result is 4 more than the number. Find the number.

   C. Find the measure of an angle whose supplement measures 10 times the measure of its complement.

   D. Find the measure of an angle such that the difference between the measures of its supplement and three times it complement is 10°.

1. Distance Problems:

   Always sketch a drawing and make a chart to organize info.

   A. Going the same direction:

      At a given hour, two steamboats leave a city in the same direction on a straight canal. One travels at 18 miles per hour and the other travels at 25 miles per hour. In how many hours will the boats be 35 miles apart?

   B. Going away from one another:

      Two trains leave a city at the same time. One travels north at 60 miles per hour and the other travels south at 80 miles per hour. In how many hours will they be 280 miles apart?

C. Going toward one another:

Atlanta and Cincinnati are 440 miles apart. John leaves Cincinnati, driving toward Atlanta at an average speed of 60 miles per hour. Pat leaves Atlanta at the same time, driving toward Cincinnati in her antique auto, averaging 28 miles per hour. How long will it take them to meet?

2. <u>Geometric Problems:</u>

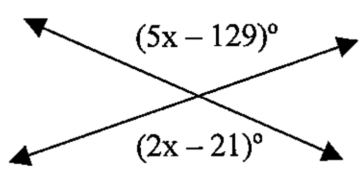
Need to decide what formula is to be used, then let x = basic unknown and design an equation.

A. A couple wishes to add a laundry room onto their house. Due to construction limitations, the length of the room must be 2 feet more than the width. Find the length and the width of the room if the perimeter is 30 feet.

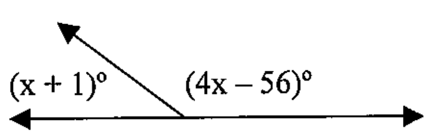
B. The largest poster ever constructed was made by the citizens of Obihiri, Hokkaido, Japan. It was in the shape of a square, and its perimeter was 1312 feet. What was the length of a side of the square poster?

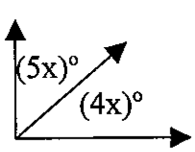
C. A puzzle piece in the shape of a triangle has a perimeter of 30 cm. Two sides of the triangle are each twice as long as the shortest side. Find the length of the shortest side.

D. Vertical angles.



e. Supplementary and complementary angles.





3. Mixture Problems:

   A. A chemist needs to mix 20 liters of 40% acid solution with some 70% solution to get a mixture that is 50% acid. How many liters of the 70% solution should be used?

   B. A certain metal is 20% tin. How many kilograms of this metal must be mixed with 80 kilograms of a metal that is 70% tin to get a metal that is 50% tin?

   C. How many gallons of a 12% indicator solution must be mixed with a 20% indicator solution to get 10 gallons of a 14% solution?

   D. Water must be added to 20 milliliters of a 4% minoxidil solution to dilute it to a 2% solution. How many milliliters of water should be used?

   E. A merchant wishes to mix candy worth \$5 per pound with 40 pounds of candy worth \$2 per pound to get a mixture that can be sold for \$3 per pound. How many pounds of \$5 candy should be used?

4. Money and Interest:

   A. Elizabeth Thornton receives an inheritance. She plans to invest part of it at 9% and 2000 more than this amount at 10%. To earn \$1150 per year in interest, how much should she invest at each rate?

   B. Walt made an extra \$10,000 last year from a part-time job. He invested part of the money at 9% and the rest at 10%. He made a total of \$930 in interest. How much was invested at 10%?

5. Monetary Value:

   A. A cashier has a total of 126 bills, made up of fives and tens. The total value of the money is \$840. How much of each kind does he have?

   B. A coin collector has \$1.70 in dimes and nickels. She has 2 more dimes than nickels. How many nickels does she have?

6. Consecutive Integers:

Consecutive	Consecutive Odd	Consecutive Even
x	x	x
x + 1	x + 2	x + 2
x + 2	x + 4	x + 4
⋮	⋮	⋮

A. When the small of two consecutive integers is added to three times the larger, the result is 59. Find the integers.

B. If the middle of three consecutive integers is added to 100, the result is 1 less than the sum of the largest and twice the smallest. Find the integers.

C. If the first and third of three consecutive even integers are added, the result is 22 less than three time the second integer. Find the integers.

D. Find two consecutive odd integers such that twice the larger is 17 more than the smaller.

7. Solving for a Specified Variable: (literal equations)

Solve as you would any equations—same steps, just variables.

A. $p = 21 + 2w$ for 1

B. $p = a + b + c$ for a

C. $f = 9/5c + 32$ for c

D. $I = \dfrac{ne}{nr + R}$ for n

E. $A = \dfrac{24f}{b(p+1)}$ for p

Work Problem:
If Joe can paint a house in 3 hours and Sam can paint the same house in 5 hours, how long does it take them to complete the job working together?

If Joe and Sam can paint the house in 5 hours working together, how long does it take Sam working alone if he takes 1 hour longer than Joe?

Modeling: Look in book for examples.

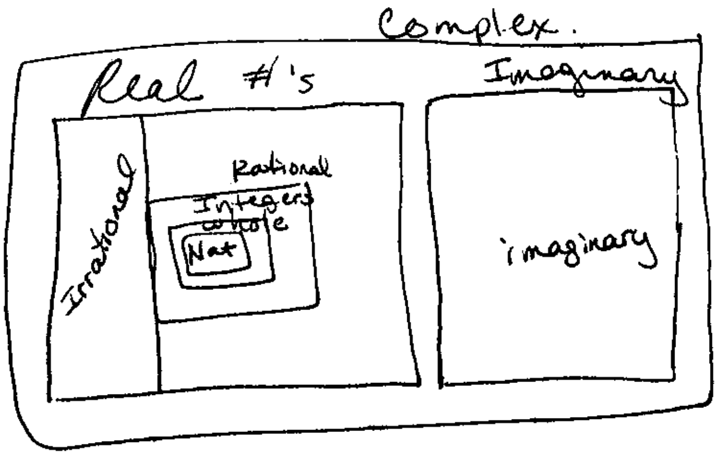
$Y = .2145x + 15.69$ models the approximate college enrollment, in millions, from fall 2003-fall 2012, where $x=0$ represents 2003, $x=1$ represents 2004, etc.
Use the model to determine the projected enrollment for Fall 2010.

1.3 Complex #'s

there is no <u>real</u> # such that $x^2 = -1$

definition: we introduce imaginary unit (i)

$$\boxed{\begin{array}{l} i^2 = -1 \\ i = \sqrt{-1} \end{array}}$$



$a + bi$:
Standard form
for complex #

① $\sqrt{-16} = \sqrt{-1} \cdot \sqrt{16} = i4$ or $\underline{4i}$

② $\sqrt{-8} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2} = 2i\sqrt{2}$

③ $(5+2i)(3-5i)$

foil

$15 \underbrace{- 25i + 6i}- 10i^2$

$15 - 19i - 10(-1)$

$15 - 9i + 10$

$\underline{25 - 9i}$

Powers of i

$i^1 = i$	$i^5 = i$	$i^9 = i$
$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$

and so on

divide out 4, and use the remainder as power of i.

ex i^{501}

$$\begin{array}{r} 125 \\ 4\overline{)501} \\ \underline{4} \\ 10 \\ \underline{8} \\ 21 \\ \underline{20} \\ 1 \end{array}$$

$= (i^4)^{125} \cdot i^1$

$1 \cdot i = i$

Name________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Identify the complex number as real or imaginary.

1) 9π 1) ______

A) Real B) Imaginary

2) $9i$ 2) ______

A) Real B) Imaginary

Write without negative radicands.

3) $\sqrt{-36}$ 3) ______

A) 6 B) $6i$ C) $-6i$ D) i

4) $\sqrt{-17} \cdot \sqrt{-17}$ 4) ______

A) -17 B) 17 C) $-17i$ D) $2i\sqrt{-17}$

Perform the indicated operation. Write the result in standard form.

5) $(8 - 6i) + (9 + 9i)$ 5) ______

A) $17 - 3i$ B) $17 + 3i$ C) $-17 - 3i$ D) $-1 + 15i$

6) $2i(5 - 7i)$ 6) ______

A) $14 + 10i$ B) $10i + 14i^2$ C) $10i - 14i^2$ D) $10i - 14$

7) $(5 - 4i)(7 + 2i)$ 7) ______

A) $43 - 18i$ B) $-8i^2 - 18i + 35$ C) $27 - 38i$ D) $43 + 18i$

Find the power of i.

8) i^{10} 8) ______

A) i B) -1 C) 1 D) $-i$

Perform the indicated operation. Write the result in standard form.

9) $\dfrac{7 + 5i}{3 - 2i}$ 9) ______

A) $\dfrac{11 - 29i}{5}$ B) $\dfrac{11 + 29i}{13}$ C) $\dfrac{31 - 1i}{13}$ D) $\dfrac{31 - 29i}{5}$

10) $\dfrac{6 + 3i}{5 + 2i}$ 10) ______

A) $\dfrac{24 - 27i}{29}$ B) $\dfrac{36 - 3i}{21}$ C) $\dfrac{24 - 3i}{21}$ D) $\dfrac{36 + 3i}{29}$

1.4

Quadratic Equations

$ax^2 + bx + c = 0$

You can solve this type of equation by

1. Factoring, if possible, then set each factor equal to zero.

2. Square root property if $b^2 = 0$ ex. $z^2 - 17 = 0$
   You take sq root of ea side

$$z^2 = 17$$
$$z = \pm\sqrt{17}$$

3. Completing the sq to make it look like #2 above.

Ex 1

$$x^2 - 4x = 8$$
$$x^2 - 4x + \underline{4} = 8 + \underline{4}$$
$$(x-2)^2 = 12 \quad \Rightarrow \quad x - 2 = \pm\sqrt{12}$$
$$x - 2 = 2\sqrt{3}$$
$$\boxed{x = \pm 2\sqrt{3} + 2}$$

Ex 2

$$2p^2 + 2p + 1 = 0$$
$$2p^2 + 2p = -1 \quad \text{You must have coefficient of 1 to complete sq.}$$
$$2\left(p^2 + p + \underline{\frac{1}{4}}\right) = -1 + \underline{2\left(\frac{1}{4}\right)}$$
$$2\left(p + \frac{1}{2}\right)^2 = -\frac{1}{2}$$
$$\left(p + \frac{1}{2}\right)^2 = -\frac{1}{2} \div 2 \quad \Rightarrow \quad \frac{\left(p+\frac{1}{2}\right)^2}{\sqrt{\left(p+\frac{1}{2}\right)^2} = \sqrt{-\frac{1}{4}}} = -\frac{1}{2} \bullet \frac{1}{2}$$
$$p + \frac{1}{2} = \pm\sqrt{-\frac{1}{4}} \quad \Rightarrow \quad p + \frac{1}{2} = \pm\frac{1}{2}i$$
$$\Rightarrow \quad p = \pm\frac{1}{2}i - \frac{1}{2}$$

4. Quadratic Formula – derived from the above process of completing the square.

For $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ This always works.

Ex

$x^2 - 4x + 2 = 0$ $a = 1, b = -4, c = 2$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} \Rightarrow \frac{4 \pm \sqrt{16-8}}{2} \Rightarrow \frac{4 \pm \sqrt{8}}{2} \Rightarrow \frac{4 \pm 2\sqrt{2}}{2} \Rightarrow 2 \pm \sqrt{2}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ What's under the radical is the discriminant.

$\boxed{b^2 - 4ac}$

1. If positive, perfect sq, then 2 rational solutions.
2. If positive, non-perfect sq, then 2 irrational solutions.
3. If zero, then 1 rational solution.
4. If negative, then 2 imaginary solutions.

You try:

Solve:

1. $p^2 = 27$
2. $m^2 + 5m = 6$
3. $3x^2 - x + 4 = 0$
4. $(3k + 1)^2 = 4$
   can use sq. rt prop.
5. $m^2 + 5m + 6 = 0$
   - will factor –

6. $8p^3 + 125 = 0$

You will want to mult. By LCD on 7 & 8 first.

7. $2 - \frac{5}{k} + \frac{2}{k^2} = 0$

8. $\frac{1}{3}x^2 + \frac{1}{6}x + \frac{1}{9} = 0$

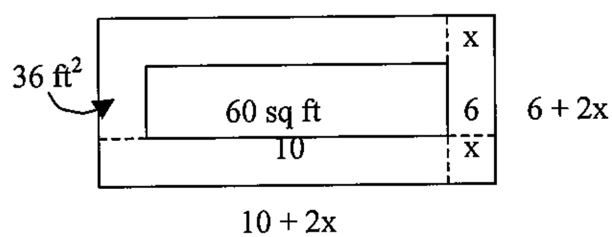
2.5 Quadratic applications

1. A landscaper wants to make an exposed gravel border of uniform width around a rectangular pool in a garden. The pool is 10 ft long and 6 ft wide. There is enough material to cover 36 ft. How wide should the border be?

x = width of border
$6 + 2x$ = width of rectangle with border added on
$10 + 2x$ = length " "

area of pool with border minus area of pool is 36 ft^2

$$(6 + 2x)(10 + 2x) - 60 = 36$$



$$60 + 12x + 20x + 4x^2 - 60 = 36$$
$$\cancel{60} + 32x + 4x^2 - \cancel{60} = 36$$
$$4x^2 + 32x - 36 = 0$$
$$x^2 + 8x - 9 = 0$$

~~60 + 32x + 4x2 – 60 = 36~~

$x + 8x - 9 = 0$
$(x + 9)(x - 1) = 0$
$x + 9 = 0$ $\quad$ $x - 1 = 0$
$\cancel{x = -9}$ $\quad$ $\boxed{x = 1}$ => The border should be 1 ft. wide.

Try:

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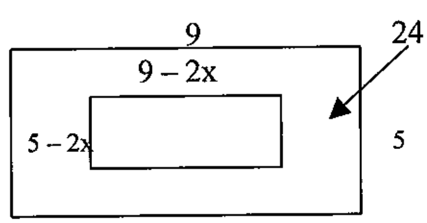


1.5 continued

The position of an object moving in a straight line is given by $s=2t^2 - 3t$, where s is in meters and t is in time in seconds the object has been in motion. How long (to the nearest tenth) will it take the object to move 14 meters?

Height of a propelled object: If air resistance is neglected, the height s (in feet) of an object propelled directly upward from an initial height of s_0 feet, with intitial velocity v_0 feet per second, is

$S = -16t^2 + v_0 t + s_0$

Where t is the number of seconds after the object is propelled. The coefficient of t^2, -16 is a constant based on the gravitational force of Earth. This constant varies on other surfaces, such as the moon and other planets.

Ex.

An astronaut on the moon throws a baseball upward. The astronaut is 6 ft. 6 in. tall and the initial velocity of the ball is 30 ft/sec. The height s of the ball in feet is given by the equation:

$S = -2.7t^2 + 30t + 6.5$ where t is the number of seconds after the ball was thrown.

A) After how many seconds is the ball 12 ft above the moon's surface?
B) How many seconds will it take for the ball to return to the surface?
C) Will the ball reach a height of 100ft? If so, after how many seconds?

Volume of a box:

A) The height of a box is 7 in. The length is 3 inches more than the width. Find the width if the volume is 910 cubic inches.

B) A rectangular piece of metal is 10 in longer than it is wide. Squares with sides 2 in. long are cut from the four corners, and the flaps are folded upward to form an open box. If the volume of the box is 832 cubic in. what were the original dimensions of the piece of metal?

quadratic applications
homework

Name________________________________

$w(8+w)=105$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The length of a rectangular frame is 8 cm more than the width. The area inside the frame is 105 square cm. Find the width of the frame.

$l = 8+w$
$w = w$

A) 7 cm B) 23 cm C) 15 cm D) 10 cm

2) The area of a square is numerically 140 more than the perimeter. Find the length of the side.

A) 98 units B) 56 units C) 14 units D) 392 units

$x^2 = 4x + 140$

3) The height of a box is 7 inches. The length is three inches more than the width. Find the width if the volume is 910.

width $= w$
length $= w+3$
$h = 7$

$7w(w+3) = 910$

A) 13 in. B) 7 in. C) 130 in. D) 10 in.

4) A rug is to fit in a room so that a border of even width is left on all four sides. If the room is 12 feet by 15 feet and the area of the rug is 108 square feet, how wide will the border be?

$(15-2x)(12-2x)=108$

A) 1.5 ft B) 2.2 ft C) 1 ft D) 3 ft

5) Two cars leave an intersection. One car travels north; the other east. When the car traveling north had gone 18 mi, the distance between the cars was 6 mi more than the distance traveled by the car heading east. How far had the eastbound car traveled?

A) 36 mi B) 24 mi C) 30 mi D) 18 mi

$x^2 + 18^2 = (x+6)^2$

6) A ladder is resting against a wall. The top of the ladder touches the wall at a height of 6 ft. Find the length of the ladder if the length is 2 ft more than its distance from the wall.

A) 10 ft B) 12 ft C) 8 ft D) 6 ft

7) A boat is 324 feet from the base of cliff. If the distance from the top of the cliff to the boat is 81 more than twice the height of the cliff. Find the height of the cliff. Round to the nearest tenth of a foot if necessary.

A) 181.1 feet B) 81 feet C) 243 feet D) 135 feet

8) A ball is dropped from a cliff that is 352 feet high. The distance S (in feet) that it falls in t seconds is given by the formula $S = 16t^2$. How many seconds (to tenths) will it take for the ball to hit the ground?

A) 18.3 sec B) 4.7 sec C) 7744 sec D) 18.8 sec

9) A rock falls from a tower that is 256 feet high. As it is falling, its height is given by the formula $h = 256 - 16t^2$. How many seconds (in tenths) will it take for the rock to hit the ground ($h = 0$)?

A) 4 sec B) 4096 sec C) 16 sec D) 15.5 sec

2. A river boat traveled upstream 12 miles. On the return trip downstream, the boat traveled 3 mph faster. If the return trip took 8 minutes less time, how fast did the boat go upstream?

$d = r \cdot t$

	d =	r	x t
up	12	x	$\frac{12}{x}$
down	12	x + 3	$\frac{12}{x+3}$

$\Bigg\} \quad t = \frac{d}{r}$

downstream = up minus 8 minutes

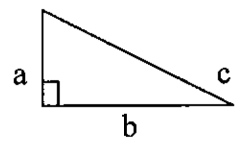
$$\frac{12}{x+3} = \frac{12}{x} - \frac{8}{60}$$

Note: you must convert to hr. because rate is mph.

Then solve.

Also note: Some problems include the current, the speed of the wind, water, etc. If so, add the current to the rate when going downstream and subtract the current from the rate going upstream.

3. Pythagorean Theorem



$a^2 + b^2 = c^2$

You try:

p132 #23, 29, 30, 34

Other Types of Equations (substitution) 1.6

1. $6p^{-2} + p^{-1} = 2$

We will use substitution.

We will let $u = p^{-1}$.

$\therefore \quad 6u^2 + u = 2$ Now solve.

$6u^2 + u - 2 = 0$

$(2u - 1)(3u + 2) = 0$

$2u - 1 = 0 \quad 3u + 2 = 0$

$u = \frac{1}{2} \quad u = -\frac{2}{3}$ Now we must change u back => unsub.

$(p^{-1})^{-1} = \left(\frac{1}{2}\right)^{-1} \quad (p^{-1})^{-1} = \left(-\frac{2}{3}\right)^{-1}$

$\boxed{p = 2} \quad \boxed{p = -\frac{3}{2}}$

Neg. exp turn fraction upside down.

now unsub

$(r-1)^{1/3}=-4$ $\quad$ $(r-1)^{1/3}=3$

$r-1=-64$ $\quad$ $r-1=27$

$r=-63$ $\quad$ $r=28$

You try:

1. $7p^{-2}+19p^{-1}=6$

2. $(p+2)^2-2(p+2)-15=0$

3. $(r-\ )^{\frac{2}{3}}+(r-1)^{\frac{1}{3}}-12=0$

   sub. $u=(r-1)^{\frac{1}{3}}$ $\therefore$ $u^2+u-12=0$

   $\vdots\,(u+4)(u-3)=0$

   $u=-4$ $\quad$ $u=3$

1.6 (Cont.) Radicals

1. $\sqrt{15-2x}=x$ Sq. both sides.

   15 – 2x = x^2

   0 = x^2 + 2x – 15

   x = –5 or x = 3

Note: You must check these; sometimes they won't work.
Sometimes both work; sometimes neither $\varnothing$.

2. $\sqrt{3-x}=\sqrt{9x}$ Square both sides.

   3 – x = 9x

   3 = 10x

   $\frac{3}{10}=x$

3. $\sqrt{2x+3}-\sqrt{x+1}=1$

(This is the situation that causes the most work—2 radicals and a constant.)

$\sqrt{2x+3}=\sqrt{x+1}+1$

$\left(\sqrt{2x+3}\right)^2=\left(\sqrt{x+1}+1\right)^2$

Isolate one radical at a time and square both sides.

$2x+3=\left(\sqrt{x+1}\right)^2+2\sqrt{x+1}+1^2$

$2x+3=x+1+2\sqrt{x+1}+1$

=> $\underline{x+1=2\sqrt{x+1}}$

$(x+1)^2=\left(2\sqrt{x+1}\right)^2$

You still have a radical—now isolate it and sq. both sides.

$x^2+2x+1=4(x+1)$

$x^2+2x+1=4x+4$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3 \quad x=-1$ (2 possible solutions to check) Both work!

<u>You try:</u>

1. $\sqrt{4r+13}=2r-1$

2. $\sqrt{5k+1}-\sqrt{3k}=1$

<u>1.6 cont</u>

Rational equations

$$\frac{2x+1}{x-2}+\frac{3}{x}=-\frac{6}{x^2-2x}$$

1.7

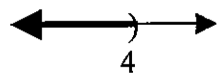
Inequalities

1. -3x + 5 > -7 (Solve as you would any linear equation except when you divide (or multiply) by a negative, turn the inequality sign around.)

$-3x+5>-7$ (Solve as you would any linear equation except when you divide (or multiply) by a negative, turn the inequality sign around.)

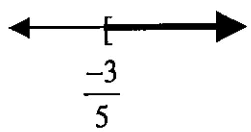
$-3x>-12$

$\frac{-3x}{-3}>\frac{-12}{-3}$ $(-\infty, 4)$

$\underline{x<4}$  x equals all the real #s less than 4 (not including 4).

2. $4-3y\le 7+2y$

$-5y\le 3$ $\left[-\frac{3}{5},\infty\right)$

$\frac{\cancel{-5}y}{\cancel{-5}}\le\frac{3}{-5}$ 

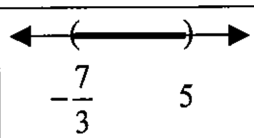
$\underline{y\ge -\frac{3}{5}}$ Greater than or equal to $-\frac{3}{5}$ $\left(-\frac{3}{5}\text{ included so use brackets}\right)$

You try:

1. $2(m+5)-3m+1\ge 5$

2. $5r+3\ge -2$

Three-part Inequality

1.	$-2<5+3x<20$		Goal is to get x alone in the middle. This will show the 2 #s that x is between. Just do the same thing to all 3 parts.
	$-7<3x<15$		
	$-\frac{7}{3}<x<5$	1. 2.	Minus 5 from all 3 parts. Divide by 3 on all 3 parts.
			$\left(-\frac{7}{3},5\right)$

You try:

1. $-3 \le \dfrac{x-4}{-5} < 4$

1.7 Quadratic Inequalities

$$ax^2 + bx + c > 0$$

You must solve these as you did quadratic equations. This will give you boundaries that you use to test regions.

Ex 1

$$x^2 - x - 12 < 0$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$

$(x-4)(x+3) < 0$

$+ \quad \vdots \quad - \quad \vdots \quad +$

$-3 \qquad 4$

Test each region to see which makes the original inequality true (less than 0) so negatives

$x - 4 = 0 \qquad x + 3 = 0$

$\underline{x = 4} \qquad \underline{x = -3}$ (boundaries)

$(-3, 4)$

$-3 \quad 4$

Ex 2

$$2x^2 + 5x - 12 \ge 0$$
$$2x^2 + 5x - 12 = 0$$
$$(2x-3)(x+4) = 0$$

$\underline{x = \dfrac{3}{2}} \qquad \underline{x = -4}$

$(2x-3)(x+4) \ge 0$ Looking for ≥ 0 (zero and positives)

$+ \quad \vdots \quad - \quad \vdots \quad +$

$-4 \qquad \dfrac{3}{2}$

$-4 \qquad \dfrac{3}{2}$

$(-\infty, -4] \cup \left[\dfrac{3}{2}, \infty\right)$

Test for ex

(–5) (0) (2)

Note: We used brackets on this because $\ge$ (this means greater than or equal to so we include the point as part of the solution, but we always use parentheses ( ) around infinity ∞).

You try:

1. $r^2 + 4r + 3 \ge 0$

2. $m^2 - 2m \le 1$

1.7

Rational Inequalities

An inequality containing a ratio (fraction)

Step 1

Rewrite the inequality, if necessary, so that zero is on one side.

Step 2

Determine what would make the denominator equal to zero; this is a boundary.

Step 3

Solve the equation to determine the solution set; this or these are boundaries. If you set the inequality equal to 0 (zero) in Step 1, then this would just be what makes the numerator equal zero.

Ex 1

$\frac{5}{x+4} \geq 1 \qquad \frac{5}{x+4} - 1 \geq 0 \qquad \frac{5}{x+4} - \frac{x+4}{x+4} \geq 0 \qquad \frac{5-(x+4)}{x+4} \geq 0$

$\frac{5-x-4}{x+4} \geq 0 \qquad \frac{1-x}{x+4} > 0$

Step 2

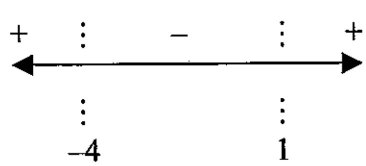
$\therefore x+4=0$ $\qquad$ $\underline{x=-4}$ boundary

Step 3

$\frac{1-x}{x+4} = \frac{0}{1}$ $\qquad$ $1(1-x) = 0(x+4)$ $\qquad$ When you cross mult., you always just get your numerator equal to zero.

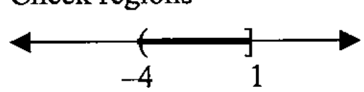
$1-x = 0$

$\underline{x = 1}$ boundary



Looking for what will make $\frac{1-x}{x+4} > 0$ greater than zero (pos. #)

Check regions



(−4, 1] -- Note: You always use a ( ) around the # that makes the denominator equal to zero, because this number cannot be used as part of the solution. It would make the expression undefined.

1.8 Absolute Value Equations & Inequalities

1. $|x| = 3 \quad \left.\begin{matrix} x = 3 \\ x = -3 \end{matrix}\right\}$ Both would make the equation true.

2. $|4x + 2| = 7$

$$\begin{array}{ll} 4x + 2 = 7 & 4x + 2 = -7 \\ 4x = 5 & 4x = -9 \\ \underline{x = \frac{5}{4}} & \underline{x = -\frac{9}{4}} \end{array}$$

3. $|3x - 2| = -6$
   Impossible $\varnothing$

4. $|x + 7| + 2 = 3$

$|x + 7| = 1$ (<u>Must</u> isolate abs. Value before you break into 2 equations.)

$$\begin{array}{ll} x + 7 = 1 & x + 7 = -1 \\ \underline{x = -6} & \underline{x = -8} \end{array}$$

5. $|x + 7| - 5 = -3$

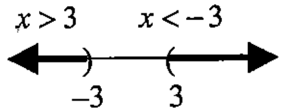
$|x + 7| = 2$

$$\begin{array}{ll} x + 7 = 2 & x + 7 = -2 \\ \underline{x = -5} & \underline{x = -9} \end{array}$$

<u>Inequalities</u>

1. $|x| > 3$

$x > 3 \qquad x < -3$



$(-\infty, -3) \cup (3, \infty)$

notice
> splits apart
< closed interval

2. $|x|<3$ (Less than 3 units from zero.)

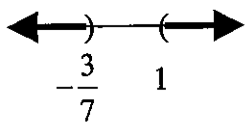
$x<3$ and $x>-3$

$(-3, 3)$

3. $|2-7m|-1>4$

$|2-7m|>5$ $\quad$ $2-7m>5$ or $2-7m<-5$

$m<-\frac{3}{7}$ $\quad$ $m>1$



4. $|4x-7|<-3$

Impossible $\varnothing$

5. $|4x-7|>-3$

Always $(-\infty, \infty)$

Absolute value = abs. Value
2 possibilities either the same numbers or opposite numbers.

$|7|=|7|$ or $|7|=|-7|$

<u>Ex 1</u>

$|2k-3|=|5k+4|$

$2k-3=5k+4$ or $2k-3=-(5k+4)$

$\left\{-\frac{7}{3}, -\frac{1}{7}\right\}$

You try:

1. $|3m-1|=2$

2. $|8-3t|-3=-2$

3. $|3-2x|=|5-2x|$

4. $|3z+1|\geq 7$

5. $\left|5x+\frac{1}{2}\right|-2<5$

ma 109 ch 2 test
Cindy Shelton

23 Sample test

Can Skip any 5

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

~~4~~ 3 pts each

Decide whether the equation is conditional, an identity, or a contradiction. Give the solution set.

1) $18m + 10 = 2(4m + 20)$

$18m+10=8m+40$

$10m=30$

$m=3$

conditional
$m=3$

2) $15k + 125 = 5(3k + 24)$

$15k+125=15k+120$

$125=120$

$\varnothing$ contradiction

Solve the equation.

3) $6[7a - 5 + 7(a + 1)] = -4a + 7$

$6[7a-5+7a+7]=-4a+7$

$6[14a+2]=-4a+7$

$84a+12=-4a+7$
$+4a \qquad -12$
$88a=-5$

$\boxed{a=-5/88}$

4) $30x\left(1 - \frac{9}{10x} = \frac{7}{3}\right)$

$30x-27=70x$

$-27=40x$

$\boxed{x=-27/40}$

Solve the formula for the indicated variable.

5) $A = P(1 + nr)$ for r

$A=P+Pnr$

$A-P=Pnr$

$\boxed{r=\frac{A-P}{Pn}}$ or $r=\frac{A}{Pn}-\frac{1}{n}$

Solve the problem.

6) Find the length of a rectangular lot with a perimeter of 76 meters if the length is 4 meters more than the width. $(P = 2L + 2W)$

length $= w+4$
width $= w$
Perimeter $= 76$

$2l+2w=P$
$2(w+4)+2w=76$
$2w+8+2w=76$
$4w+8=76$
$4w=68$

$w=17$
$+4$
21

7) Martha can rake the leaves in her yard in 8 hours. Her brother can do the job in 3 hours. How long will it take them to do the job working together?

$24\left(\frac{1}{8}\cdot t + \frac{1}{3}\cdot t = 1\right)$

$3t+8t=24$

$11t=24$

$t=24/11$

x^2-5x-6

8) In a chemistry class, 6 liters of a 4% silver iodide solution must be mixed with a 10% solution to get a 6% solution. How many liters of the 10% solution are needed?

$\boxed{6 \;\; 4\%} + \boxed{x \;\; 10\%} = \boxed{x+6 \;\; 6\%}$

$.04(6) + .10(x) = .06(x+6)$

$4(6) + 10x = 6(x+6)$

$24 + 10x = 6x + 36$

$4x = 12$

$x = 3$

Perform the indicated operation. Write the result in standard form.

9) $\frac{7+6i}{9-2i} \cdot \frac{(9+2i)}{(9+2i)} = \frac{63 + 14i + 54i + 12i^2}{81 - 4i^2} = \frac{63 + 68i - 12}{81+4}$

$= \frac{51 + 68i}{85} = \frac{17(3+4i)}{85} = \boxed{\frac{3+4i}{5}}$

Solve the equation by completing the square.

10) $t^2 + 10t + 4 = 0$

$t^2 + 10t + \underline{25} = -4 + 25$

$\sqrt{(t+5)^2} = \sqrt{21}$

$t + 5 = \pm\sqrt{21}$

$\boxed{t = \pm\sqrt{21} - 5}$ or $\boxed{-5 \pm \sqrt{21}}$

Solve the equation.

11) $\sqrt{(x+13)^2} = \sqrt{4}$

$t + 13 = \pm 2$

$t + 13 = 2$ $\quad \boxed{t = -11}$

$t + 13 = -2$ $\quad \boxed{t = -15}$

12) $k^2 - 8k - 3 = 0$

$a = 1$

$b = -8$

$c = -3$

$\frac{8 \pm \sqrt{64 - 4(1)(-3)}}{2(1)} = \frac{8 \pm \sqrt{64+12}}{2} = \frac{8 \pm \sqrt{76}}{2} = \frac{8 \pm \sqrt{4 \cdot 19}}{2}$

$\frac{8 \pm 2\sqrt{19}}{2}$

$\boxed{4 \pm \sqrt{19}}$

Evaluate the discriminant $b^2 - 4ac$ and predict the type and number of solutions.

13) $s^2 - 6s + 1 = 0$

$a = 1$

$b = -6$

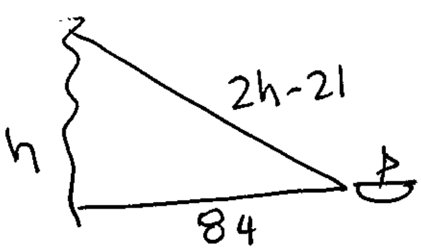
$c = 1$

$36 - 4(1)(1)$

$36 - 4 = $ $\boxed{\text{32, two diff. irrational}}$

Solve the problem.

14) A boat is 84 feet from the base of cliff. If the distance from the top of the cliff to the boat is 21 less than twice the height of the cliff to the water. Find the height of the cliff. Round to the nearest tenth of a foot if necessary.



$h^2 + 84^2 = (2h-21)^2$

$h^2 + 7056 = 4h^2 - 84h + 441$

$0 = 3h^2 - 84h - 6615$

$0 = 3(h^2 - 28h - 2205)$

$0 = (h-63)(h+35)$

$h = 63$ $\quad$ ~~h = -35~~

15) A rock falls from a tower that is 160 feet high. As it is falling, its height is given by the formula $h = 160 - 16t^2$. How many seconds (in tenths) will it take for the rock to hit the ground ($h = 0$)?

160

$h = 160 - 16t^2$

$0 = 160 - 16t^2$

$16t^2 - 160 = 0$

$16(t^2 - 10) = 0$

$t^2 - 10 = 0$

$\sqrt{t^2} = \sqrt{10}$

$t = \pm\sqrt{10}$

$\boxed{t \approx 3.2 \text{ sec}}$

Solve the equation.

16) $\sqrt{2x+3} - \sqrt{x+1} = 1$

$(\sqrt{2x+3})^2 = (\sqrt{x+1} + 1)^2$

$2x+3 = x+1 + 2\sqrt{x+1} + 1$

$(x+1)^2 = (2\sqrt{x+1})^2$

$x^2 + 2x + 1 = 4(x+1)$

$x^2 + 2x + 1 = 4x + 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3 \quad x = -1}$

both work

17) $\sqrt{x^2 - 3} - \sqrt{x+3} = 0$

$(\sqrt{x^2-3})^2 = (\sqrt{x+3})^2$

$x^2 - 3 = x + 3$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x = 3 \quad x = -2}$

both work

Solve the equation using the method of substitution.

18) $4p^{-2} + 3p^{-1} - 10 = 0$

let $u = p^{-1}$

$4u^2 + 3u - 10 = 0$

$(4u-5)(u+2)$

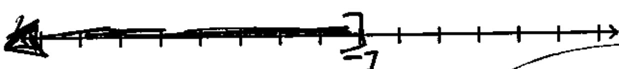
$u = 5/4 \qquad u = -2$

$p^{-1} = 5/4 \qquad p^{-1} = -2$

$\boxed{p = \frac{4}{5} \quad p = -\frac{1}{2}}$

Solve and graph the inequality. Give answer in interval notation.

19) $-13a - 12 \geq -12a - 5$



$-13a - 12 \geq -12a - 5$

$+12a \qquad\qquad +12$

$-a \geq 7$

$a \leq -7$

$(-\infty, -7]$

Solve the inequality and graph the solution set.

20) $t^2 - 4t - 12 \le 0$

$(t-6)(t+2) = 0$

$t = 6$

$t = -2$

-2 $\quad$ 6

$[-2, 6]$

Solve the inequality. Write answer in interval notation.

21) $\frac{-5x + 4}{3} > 0$

$\frac{-5x+4}{3} = \frac{0}{1}$

$0 = -5x + 4$

$x = +4/5$ only bound.

$4/5$

$\boxed{(-\infty, 4/5)}$

22) $\frac{4x}{7 - x} < x$

$\frac{4x}{7-x} = \frac{x}{1}$

$4x = x(7-x)$

$4x = 7x - x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\underline{x = 0}$ $\quad$ $\underline{x = 3}$

+ what makes denom = zero.

$7 - x = 0$

$\underline{7 = x}$

$0 \quad 3 \quad 7$

$\boxed{(0, 3) \cup (7, \infty)}$

Graph the solution set.

23) $|x| = 1$

$-1 \quad 1$

$\boxed{x = 1 \qquad x = -1}$

Solve the equation.

24) $|7m + 8| + 2 = 7$

$|7m + 8| = 5$

$7m + 8 = 5$

$\boxed{m = -3/7}$

$7m + 8 = -5$

$\boxed{m = -13/7}$

25) $|4s - 6| = |s + 7|$

$4s - 6 = s + 7$

$3s = 13$

$\boxed{s = 13/3}$

&

$4s - 6 = -(s+7)$

$4s - 6 = -s - 7$

$5s = -1$

$\boxed{s = -1/5}$

26) $|7x - 9| = -12$

$\emptyset$

Solve the inequality.

27) $|4k + 1| + 9 < 11$

$|4k+1| < 2$

$4k+1 < 2$ & $4k+1 > -2$

$4k < 1$ $4k > -3$

$k < 1/4$ $k > -3/4$

$-3/4$ $1/4$

$(-3/4, 1/4)$

28) $|-9 + 3x| > 2$

$-9+3x > 2$ & $-9+3x < -2$

$3x > 11$ $3x < 7$

$x > 11/3$ $x < 7/3$

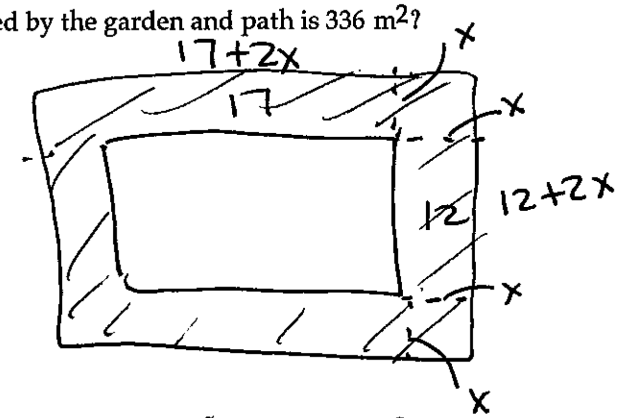
$7/3$ $11/3$

$(-\infty, 7/3) \cup (11/3, \infty)$

Solve the problem.

29) A 12 m by 17 m rectangular garden is to have a gravel path of uniform width bordering it. How wide is the path if the total area covered by the garden and path is 336 m^2?

bonus: show all work



$(17+2x)(12+2x) = 336$

$204 + 34x + 24x + 4x^2 = 336$

$4x^2 + 58x - 132 = 0$

$2(2x^2 + 29x - 66) = 0$

$2(2x+33)(x-2) = 0$

$2x+33=0$ $x-2=0$

$\cancel{x = -33/2}$ $x=2$